

AIRPORT SENIOR SECONDARY SCHOOL
TERMINAL EXAMINATION-1 (2023-24)

MATHEMATICS

Class: XII

Marks: 80

Time: 3Hrs

SECTION A

1. If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, & \text{when } i \neq j \\ 0, & \text{when } i = j \end{cases}$, then A^2 is
- (a) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}_{2 \times 2}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}_{2 \times 2}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}_{2 \times 2}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2}$
2. If A and B are invertible square matrices of the same order, then which of the following is not correct?
- (a) $|\mathbf{AB}^{-1}| = \frac{|\mathbf{A}|}{|\mathbf{B}|}$ (b) $|(\mathbf{AB})^{-1}| = \frac{1}{|\mathbf{A}| |\mathbf{B}|}$
- (c) $(\mathbf{AB})^{-1} = \mathbf{B}^{-1} \mathbf{A}^{-1}$ (d) $(\mathbf{A} + \mathbf{B})^{-1} = \mathbf{B}^{-1} + \mathbf{A}^{-1}$
3. If the area of the triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 sq units, then the value/s of k will be
- (a) 9 (b) ± 3 (c) -9 (d) 6
4. If $f(x) = \begin{cases} kx, & \text{if } x < 0 \\ |x|, & \text{if } x \geq 0 \end{cases}$ is continuous at $x = 0$, then the value of k is
- (a) -3 (b) 0 (c) 3 (d) any real number
- 5.
- . Given that A is a square matrix of order 3 and $|A| = -2$, then $|\text{adj}(2A)|$ is equal to
- (a) -2^6 (b) +4 (c) -2^8 (d) 2^8
- 6.
- . The set of all points where the function $f(x) = x + |x|$ is differentiable, is
- (a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-\infty, \infty)$
- 7.

If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then the possible value(s) of 'x' is/are
 (a) 3 (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) $\sqrt{3}, -\sqrt{3}$

8.

If $y = \sin^{-1}x$, then $(1 - x^2)y_2$ is equal to
 (a) xy_1 (b) xy (c) xy_2 (d) x^2

9.

The function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3 - 4x$ is

(a) one-one (b) onto (c) one-one and onto (d) none of these

10.

If R is a relation in a set A such that $(a, a) \in R$ for every $a \in A$, then the relation R is called

(a) reflexive (b) symmetric (c) transitive (d) none of these

11.

If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ then value of x is
 (a) 3 (b) ± 6 (c) 8 (d) -2

12.

The principal value of $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is

(a) $\frac{\pi}{3}$ (b) $2\pi/3$ (c) $3\pi/2$ (d) $\pi/2$

13.

$A = [a_{ij}]_{m \times n}$ is a square matrix if

(a) $m < n$ (b) $m > n$ (c) $m = n$ (d) none of these

14.

If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(a) A (b) $I - A$ (c) I (d) 3A

15.

If area of a triangle is 35 sq units with vertices (2,-6), (5,4) and (k,4), then the value of k is

(a) 12 (b) -2 (c) -12, -2 (d) 12, -2

16.

The derivative of $\sin(\log x)$ is

- (a) $\cos(\log x)$ (b) $\sin(\log x)$ (c) $\frac{\cos(\log x)}{x}$ (d) $\frac{\sin(\log x)}{x}$

17.

The rate of change of area of a circle with respect to its radius r at $r=6$ cm is

- (a) 10π (b) 12π (c) 8π (d) 11π

18.

The interval in which $x^2 e^{-x}$ is increasing is

- (a) $(-\infty, \infty)$ (b) $(2, \infty)$ (c) $(-2, 0)$ (d) $(0, 2)$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

Q19. Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}(f(x)) = (x-1)^3(x-3)^2$, then

ASSERTION (A): $f(x)$ has a minimum at $x = 1$.

REASON (R): When $\frac{d}{dx}(f(x)) < 0, \forall x \in (a-h, a)$ and $\frac{d}{dx}(f(x)) > 0, \forall x \in (a, a+h)$; where ' h ' is an infinitesimally small positive quantity, then $f(x)$ has a minimum at $x = a$, provided $f(x)$ is continuous at $x = a$.

Q20. ASSERTION (A): The relation $f: \{1, 2, 3, 4\} \rightarrow \{x, y, z, p\}$ defined by $f = \{(1, x), (2, y), (3, z)\}$ is a bijective function.

REASON (R): The function $f: \{1, 2, 3\} \rightarrow \{x, y, z, p\}$ such that $f = \{(1, x), (2, y), (3, z)\}$ is one-one.

SECTION B

21. Find the value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$.

OR

Find the domain of $\sin^{-1}(x^2 - 4)$.

22. Find the interval/s in which the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = xe^x$, is increasing.

23. If $f(x) = \frac{1}{4x^2 + 2x + 1}$; $x \in \mathbb{R}$, then find the maximum value of $f(x)$.

OR

Find the maximum profit that a company can make, if the profit function is given by

$P(x) = 72 + 42x - x^2$, where x is the number of units and P is the profit in rupees.

24. Find $\frac{dy}{dx}$ if $3x + 2y = \sin y$

25. Check whether the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3 + x$, has any critical point/s or not?

If yes, then find the point/s.

SECTION C

26. Show that the relation R in the set Z of integers given by $R = \{(a,b): 3 \text{ divides } a-b\}$ is an equivalence relation.

27.

If the function $f(x)$ given by

$$f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$$

Is continuous at $x=1$ find the value of a and b .

28. Find $\frac{dy}{dx}$ if $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$.

29. If $y = A \sin x + B \cos x$, then prove that $\frac{d^2y}{dx^2} + y = 0$

30. The volume of a cone is increasing at the rate of 8 cu. cm /sec. How fast is the surface area increasing when the length of the edge is 11 cm.

31. Find the interval in which the function $f(x) = 2x^3 - 3x^2 - 36x + 15 = 0$ is (a) strictly increasing (b) strictly decreasing.

SECTION D

32. Differentiate $(x \cos x)^x + (x \sin x)^{1/x}$

33. If $y = \sin^{-1}x$, show that $(1 - x^2) y_2 - x y_1 = 0$

34. Find the absolute maximum value and the absolute minimum value of the function $f(x) = \sin x + \cos x$ in the interval $[0, \pi]$.

35. Prove that the largest cone that can be inscribed in a sphere of radius R is $8/27$ of the volume of the sphere.

SECTION E

36.



Two farmers Ram Narayan and Gurbacchan Singh cultivate only three variety of rice namely Basmati, Parmal and Naura . The sale in Rupees of three variety of rice by both of the Farmers in the month of September and October are given by given by the following matrices A and B .

September sale(in Rupees)

Basmati Parmal Naura

$$A = \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \quad \begin{array}{l} \text{Ram Narayan} \\ \text{Gurbacchan Singh} \end{array}$$

October sale(in Rupees)

Basmati Parmal Naura

$$B = \begin{bmatrix} 50000 & 10000 & 60000 \\ 20000 & 10000 & 10000 \end{bmatrix} \quad \begin{array}{l} \text{Ram Narayan} \\ \text{Gurbacchan Singh} \end{array}$$

On the basis of above information answer the following questions:

- (i) Find the combined sale in September & October for each farmer in each variety .
- (ii) If both farmers receive 2% profit on gross sale . Compute the profit for each farmer for each variety sold in October.

37.

$P(x) = -5x^2 + 125x + 37500$ is the total profit function of a company, where x is the production of the company.



1. What will be the production when the profit is maximum?
2. What will be the maximum profit?
3. Check in which interval the profit is strictly increasing .
4. When the production is 2units what will be the profit of the company?

38. Read the following passage and answer the questions given below:

The relation between the height of the plant ('y' in cm) with respect to its exposure to the sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$, where 'x' is the number of days exposed to the sunlight, for $x \leq 3$.



- (i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.

- (ii) Does the rate of growth of the plant increase or decrease in the first three days?
What will be the height of the plant after 2 days?
